



Cambridge International AS & A Level

CANDIDATE
NAME

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

2

- 1** Find the Maclaurin's series for $\tan\left(x + \frac{1}{4}\pi\right)$ up to and including the term in x^2 . [5]

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- 2** A curve has equation $y = \cosh x$, for $0 \leq x \leq \frac{1}{2}$.
Find, in terms of π and e, the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [6]

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- 3 Find all the roots of the equation $(w + 1)^6 = 1$, giving your answers in the form $x + iy$ where x and y are real and exact. [4]

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5 The curve C has equation

$$y^2 + (xy + 1)^2 = 5.$$

- (a) Show that, at the point $(1, 1)$ on C , $\frac{dy}{dx} = -\frac{2}{3}$. [3]

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- (b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 1)$. [5]

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- 6 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102 \cos 3t,$$

given that, when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 0$. [11]

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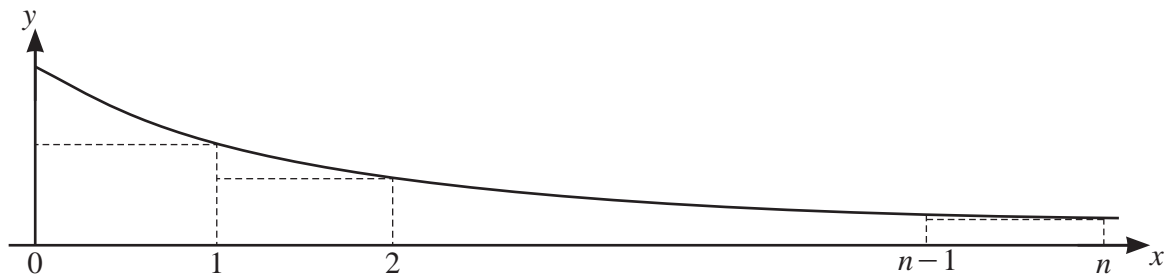
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The diagram shows the curve $y = \frac{1}{\sqrt{x^2 + x + 1}}$ for $x \geq 0$, together with a set of n rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r^2 + r + 1}} < \ln\left(\frac{1}{3} + \frac{2}{3}n + \frac{2}{3}\sqrt{n^2 + n + 1}\right). \quad [10]$$

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The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

- (b) Show that the eigenvalues of \mathbf{A} are a , -1 and -4 . [2]

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- (c) Find a matrix \mathbf{P} such that

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \mathbf{P}^{-1}. \quad [5]$$

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(d) Use the characteristic equation of **A** to find \mathbf{A}^{-1} . [6]

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